To: CIHR Workshop Attendees
From: P. A. Jansson

Re: Preparation for Deconvolution Lecture

I am honored by the invitation to address your group. To best take advantage of the material presented, it would be helpful to work an exercise. I will connect the exercise to microscopy during the talk.

You can work the exercise in columns on a sheet of paper, on a spreadsheet like EXCEL, or with a simple computer program.

You are probably familiar with the concept of a weighted average. We are going to compute a simple moving weighted average of a list of sample values. Then, temporarily discarding the original sample values, from the resulting list of weighted average numbers, we will try to recover them.

Below we see columns containing indices, sample values, weights, and a single weighted average of the first 9 values, recorded in the fourth column and opposite the largest weight. The weighted average is obtained by multiplying each of the nine sample values opposite each of the weights by their respective weights, then adding those products together. Because the first nine values are zero, the sum is zero. For this particular set of weights, the *sum* is the same as the *average* because the weights are *normalized*, *i.e.,* adding them together gives unity.

```
1            2                  3                              4

Sample       Sample                                            Weighted
Indices      Values             Weights                        Average

1            0         x        0.04   =     0      \
2            0         x        0.08   =     0      |
3            0         x        0.12   =     0      |(sum)
4            0         x        0.16   =     0      |
5            0         x        0.20   =     0       >  =      0
6            0         x        0.16   =     0      |
7            0         x        0.12   =     0      |
8            0         x        0.08   =     0      |
9            0         x        0.04   =     0      /
10           0
11           2                    |
12           0                    |
13           3                    | (slide down)
14           0                    V
15           0
16           0
17           0
18           0
19           0
20           0
21           0
22           0
23           0
24           0
25           0
26           0
```

Now, slide the nine weights down so that they are opposite sample values 2 through 10, and repeat the process. You will note that after the second sliding, the bottom weight has encountered the sample having a value of 2, so it then contributes to the weighted average.

When the bottom weight encounters sample 26, you are finished, and should have 18 numbers in the weighted-average column.

At this point it will be instructive to plot the original sample values and weighted averages on a "y" axis *vs.* the sample indices on the "x" axis.

Now, here is the part where you have to think. Suppose the original sample values are unknown or inaccessible to us. Let us try to recover those values, starting from the top of the weighted-average column. We are permitted to assume that the first eight values are zeroes. Because the weighted average of the first nine values is also zero, the ninth value must be zero too. Now it is up to you to consider the second weighted-average to see if you can infer the *next* original sample value on the list, having determined the previous one, and so forth. A little algebra will permit you to develop a formula for each new unknown sample value in terms of the previous values and the newly introduced weighted-average number. By this means recover and *plot* all the original sample values.

Were the sample values recovered correctly? (Hopefully "yes," if you carried enough precision in your calculations.)

Now, add a small error, 0.02, to the weighted-average number opposite sample index 11, then repeat the recovery process, plotting the result. What did you observe?

Examples like this are contained in the introductory chapter of my text *Deconvolution of Images and Spectra*, Academic Press, 1997. This volume goes on to treat the mathematics and practical applications of deconvolution. For those who are sufficiently interested, this volume should be accessible to anyone with a background in elementary calculus. It and/or its first edition, *Deconvolution, with Applications in Spectroscopy*, Academic Press, 1984, can be found in nearly all major university libraries. Descriptions of both texts can be found by searching for "deconvolution" on the AP website:

http://www.bhusa.com/apcatalog/.

A capsule review can be found on Amazon:

http://www.amazon.com/exec/obidos/tg/detail/-/0123802229/qid=1045710093/sr=8-1/ref=sr_8_1/002-4910755-3291234?v=glance&s=books&n=507846.

See you soon!